PHY130: HW_12 Help

Solution or Explanation

(a) Taking to the right as positive, the spring force acting on the block at the instant of release is

$$F_s = -kA = -(133 \text{ N/m})(+0.13 \text{ m}) = -17.3 \text{ N or } 17.3 \text{ N to the left.}$$

(b) At this instant, the acceleration is

$$a = \frac{F_s}{m} = \frac{-17.3 \text{ N}}{0.57 \text{ kg}} = -30.3 \text{ m/s}^2 \text{ or } a = 30.3 \text{ m/s}^2 \text{ to the left.}$$

Solution or Explanation

(a) $k = \frac{F_{max}}{x_{max}} = \frac{205 \text{ N}}{0.450 \text{ m}} = 456 \text{ N/m}$

(b) work done $= PE_s = \frac{1}{2}kx^2 = \frac{1}{2}(456 \text{ N/m})(0.450 \text{ m})^2 = 46.1 \text{ J}$

Solution or Explanation

An unknown quantity of mechanical energy is converted into internal energy during the collision. Thus, we apply conservation of momentum from just before to just after the collision and obtain $mv_i + M(0) = (M + m)V$, or the speed of the block and embedded bullet just after collision is $V = (m/(M + m))v_i$. We now use conservation of mechanical energy, $(KE + PE_s)_f = (KE + PE_s)_i$, from just after the collision until the block comes to rest. This gives

$$0 + \frac{1}{2}kx_f^2 = \frac{1}{2}(M + m)V^2 + 0,$$

or

$$x_f = V\sqrt{\frac{M + m}{k}} = v_i\left(\frac{m}{M + m}\right)\sqrt{\frac{M + m}{k}} = \frac{mv_i}{\sqrt{(M + m)k}}$$

yielding

$$x_f = \frac{(10.0 \times 10^{-3} \text{ kg})(300 \text{ m/s})}{\sqrt{(1.76 \text{ kg})(16.7 \text{ N/m})}} = 0.553 \text{ m.}$$

Solution or Explanation

(a) $f = \frac{1}{T} = \frac{1}{0.582\text{ s}} = 1.72\text{ Hz}$

(b) The period of oscillation of an object-spring system is $T = 2\pi\sqrt{m/k}$, so the force constant is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.260\text{ kg})}{(0.582\text{ s})^2} = 30.3\text{ N/m}.$$

(c) At turning points ($x = \pm A$) in the oscillation, all of the energy is temporarily stored as elastic potential energy, of $E = kA^2/2$. Thus,

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.293\text{ J})}{30.3\text{ N/m}}} = 0.139\text{ m}.$$

Solution or Explanation

(a) At the equilibrium position, the total energy of the system is in the form of kinetic energy and $\frac{1}{2}mv_{max}^2 = E$, so the maximum speed is

$$v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(5.67\text{ J})}{337\text{ kg}}} = 5.80\text{ m/s}.$$

(b) The period of an object-spring system is $T = 2\pi\sqrt{m/k}$, so the force constant of the spring is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(337\text{ kg})}{(0.270\text{ s})^2} = 183\text{ N/m}.$$

(c) At the turning points, $x = \pm A$, the total energy of the system is in the form of elastic potential energy, or $E = \frac{1}{2}kA^2$, giving the amplitude as

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(5.67\text{ J})}{183\text{ N/m}}} = 0.249\text{ m}.$$

Solution or Explanation

The wavelength (and size of smallest detectable insect) is calculated as follows, using the equation $v = \lambda f$.

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{89600\text{ Hz}} = \underline{3.83\text{ mm}}$$

Solution or Explanation

(a) The mass per unit length is

$$\mu = \frac{m}{L} = \frac{0.0600\text{ kg}}{5.80\text{ m}} = 1.03 \times 10^{-2}\text{ kg/m}.$$

From $v = \sqrt{F/\mu}$, the required tension in the string is

$$F = v^2\mu = (43.0\text{ m/s})^2(1.03 \times 10^{-2}\text{ kg/m}) = 19.1\text{ N}.$$

(b) $$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{8.00\text{ N}}{1.03 \times 10^{-2}\text{ kg/m}}} = 27.8\text{ m/s}$$

Solution or Explanation

(a) Constructive interference produces the maximum amplitude.

$$A'_{max} = A_1 + A_2 = 0.50\text{ m} + 0.19\text{ m} = 0.690\text{ m}$$

(b) Destructive interference produces the minimum amplitude.

$$A'_{min} = A_1 - A_2 = 0.50\text{ m} - 0.19\text{ m} = 0.310\text{ m}$$